	Unconstrained Optimization	**Constrained Optimization**
	$\min_{x \in \mathbb{R}^n} f(x)$	$\min_{x \in \mathbb{R}^n} f(x)$ s.t. $h(x) = 0$, $g(x) \leq 0$
First Order Conditions	$\nabla f(x^*) = 0$	$0 = \nabla_x L(x^*, u^*, v^*) = \nabla f(x^*) + \nabla g(x^*)^T u^* + \nabla h(x^*)^T v^*$ "balance of forces"
Feasibility	N.A.	$g(x^*) \leq 0, h(x^*) = 0$
Complementarity	N.A.	$u^* \geq 0$ ("fence can only push in one direction") and $g(x^*)^T u^* = 0$ which implies: $g_i(x^*) = 0$ ("ball is on the fence") OR $u_i^* = 0$ ("fence does not push against the ball")
Constraint Qualification	N.A.	We will focus on LICQ and MFCQ
Second Order Conditions (SOC)	$\nabla^2 f(x^*)$ is P.S.D., i.e., $p^T \nabla^2 f(x^*) p \geq 0$, for all $p \in \mathbb{R}^n$	$p^T \nabla_{xx} L(x^*, u^*, v^*) p \geq 0$ for all $p \neq 0$ that satisfy[1]: $\nabla h(x^*)^T p = 0$; $\nabla g_i(x^*)^T p = 0, i \in \{i \mid g_i(x^*) = 0, u_i^* > 0\}$ (strongly active[2]); $\nabla g_i(x^*)^T p \leq 0, i \in \{i \mid g_i(x^*) \leq 0, u_i^* = 0\}$ (inactive[3] or weakly active[4])

[1] SOC are <u>vacuously satisfied</u> if there are no search directions, i.e., no p satisfies the three conditions involving $\nabla(h^*)$ and $\nabla(g^*)$.

[2] strongly active: $g_i(x^*) = 0$ and $u_i^* > 0$. "ball is touching the fence and the fence is pushing back against the ball".

[3] inactive: $g_i(x^*) < 0$ and $u_i^* = 0$. "ball is on the feasible side of the fence but is NOT touching the fence"

[4] weakly active: $g_i(x^*) = 0$ and $u_i^* = 0$. "ball is BARELY touching the fence and the fence is NOT pushing back"