Farmer's Problem from *Introduction to Stochastic Programming* by Birdge and Louveaux (2011)

Consider a European farmer who specializes in raising wheat, corn, and sugar beets on his 500 acres of land. During the winter, he wants to decide how much land to devote to each crop. (We refer to the farmer as "he" for convenience and not to imply anything about the gender of European farmers.)

The farmer knows that at least 200 tons (T) of wheat and 240 T of corn are needed for cattle feed. These amounts can be raised on the farm or bought from a wholesaler. Any production in excess of the feeding requirement would be sold. Over the last decade, mean selling prices have been \$170 and \$150 per ton of wheat and corn, respectively. The purchase prices are 40% more than this due to the wholesaler's margin and transportation costs.

Another profitable crop is sugar beet, which he expects to sell at \$36/T; however, the European Commission imposes a quota on sugar beet production. Any amount in excess of the quota can be sold only at \$10/T. The farmer's quota for next year is 6000 T.

Based on past experience, the farmer knows that the mean yield on his land is roughly 2.5 T, 3 T, and 20 T per acre for wheat, corn, and sugar beets, respectively. Table 1 summarizes these data and the planting costs for these crops.

To help the farmer make up his mind, we can set up the following model. Let

x_1 = acres of land devoted to wheat,
x_2 = acres of land devoted to corn,
x_3 = acres of land devoted to sugar beets,
w_1 = tons of wheat sold,
y_1 = tons of wheat purchased,
w_2 = tons of corn sold,
y_2 = tons of corn purchased,
w_3 = tons of sugar beets sold at the favorable price,
w_4 = tons of sugar beets sold at the lower price.

Table 1 Data for farmer's problem.

	Wheat	Corn	Sugar Beets
Yield (T/acre)	2.5	3	20
Planting cost (\$/acre)	150	230	260
Selling price (\$/T)	170	150	36 under 6000 T
			10 above 6000 T
Purchase price (\$/T)	238	210	–
Minimum requirement (T)	200	240	–
Total available land: 500 acres			

Perfect Information

Table 2 Optimal solution based on expected yields.

Culture	Wheat	Corn	Sugar Beets
Surface (acres)	120	80	300
Yield (T)	300	240	6000
Sales (T)	100	–	6000
Purchase (T)	–	–	–
Overall profit: $118,600			

Table 3 Optimal solution based on above average yields (+ 20%).

Culture	Wheat	Corn	Sugar Beets
Surface (acres)	183.33	66.67	250
Yield (T)	550	240	6000
Sales (T)	350	–	6000
Purchase (T)	–	–	–
Overall profit: $167,667			

Table 4 Optimal solution based on below average yields (−20%).

Culture	Wheat	Corn	Sugar Beets
Surface (acres)	100	25	375
Yield (T)	200	60	6000
Sales (T)	–	–	6000
Purchase (T)	–	180	–
Overall profit: $59,950			

Under Uncertainty

$$
\begin{aligned}
\min\ & 150x_1 + 230x_2 + 260x_3 \\
& - \tfrac{1}{3}(170w_{11} - 238y_{11} + 150w_{21} - 210y_{21} + 36w_{31} + 10w_{41}) \\
& - \tfrac{1}{3}(170w_{12} - 238y_{12} + 150w_{22} - 210y_{22} + 36w_{32} + 10w_{42}) \\
& - \tfrac{1}{3}(170w_{13} - 238y_{13} + 150w_{23} - 210y_{23} + 36w_{33} + 10w_{43}) \\
\text{s.t. } & x_1 + x_2 + x_3 \le 500\,,\ 3x_1 + y_{11} - w_{11} \ge 200\,, \\
& 3.6x_2 + y_{21} - w_{21} \ge 240\,,\ w_{31} + w_{41} \le 24x_3\,,\ w_{31} \le 6000\,, \\
& 2.5x_1 + y_{12} - w_{12} \ge 200\,,\ 3x_2 + y_{22} - w_{22} \ge 240\,, \\
& w_{32} + w_{42} \le 20x_3\,,\ w_{32} \le 6000\,,\ 2x_1 + y_{13} - w_{13} \ge 200, \\
& 2.4x_2 + y_{23} - w_{23} \ge 240,\ w_{33} + w_{43} \le 16x_3\,, \\
& w_{33} \le 6000,\ x, y, w \ge 0\,.
\end{aligned}
\tag{1.2}
$$

Table 5 Optimal solution based on the stochastic model (1.2).

		Wheat	Corn	Sugar Beets
First Stage	Area (acres)	170	80	250
$s = 1$ Above	Yield (T)	510	288	6000
	Sales (T)	310	48	6000 (favor. price)
	Purchase (T)	–	–	–
$s = 2$ Average	Yield (T)	425	240	5000
	Sales (T)	225	–	5000 (favor. price)
	Purchase (T)	–	–	–
$s = 3$ Below	Yield (T)	340	192	4000
	Sales (T)	140	–	4000 (favor. price)
	Purchase (T)	–	48	–
Overall profit: $108,390				